Please check the examination details below before entering your candidate information

Candidate surname	Other names

Centre Number

Candidate Number

Pearson Edexcel International GCSE

Friday 26 May 2023

Afternoon (Time: 2 hours)

Paper reference **4PM1/01R**

Further Pure Mathematics
PAPER 1R

Calculators may be used.

Total Marks

Instructions

- Use **black** ink or ball-point pen.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions.
- Without sufficient working, correct answers may be awarded no marks.
- Answer the questions in the spaces provided – *there may be more space than you need*.
- You must **NOT** write anything on the formulae page. Anything you write on the formulae page will gain NO credit.

Information

- The total mark for this paper is 100.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Check your answers if you have time at the end.

Turn over ▶

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Pearson

International GCSE in Further Pure Mathematics Formulae sheet

Mensuration

Surface area of sphere = $4\pi r^2$

Curved surface area of cone = $\pi r \times$ slant height

Volume of sphere = $\frac{4}{3}\pi r^3$

Series

Arithmetic series

Sum to n terms, $S_n = \frac{n}{2}[2a + (n-1)d]$

Geometric series

Sum to n terms, $S_n = \frac{a(1-r^n)}{(1-r)}$

Sum to infinity, $S_\infty = \frac{a}{1-r}$ $\quad |r| < 1$

Binomial series

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \ldots + \frac{n(n-1)\ldots(n-r+1)}{r!}x^r + \ldots \quad \text{for} \quad |x| < 1, n \in \mathbb{Q}$$

Calculus

Quotient rule (differentiation)

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{\mathrm{f}(x)}{\mathrm{g}(x)}\right) = \frac{\mathrm{f}'(x)\mathrm{g}(x) - \mathrm{f}(x)\mathrm{g}'(x)}{[\mathrm{g}(x)]^2}$$

Trigonometry

Cosine rule

In triangle ABC: $a^2 = b^2 + c^2 - 2bc\cos A$

$\tan\theta = \frac{\sin\theta}{\cos\theta}$

$\sin(A+B) = \sin A\cos B + \cos A\sin B$ $\qquad \sin(A-B) = \sin A\cos B - \cos A\sin B$

$\cos(A+B) = \cos A\cos B - \sin A\sin B$ $\qquad \cos(A-B) = \cos A\cos B + \sin A\sin B$

$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A\tan B}$ $\qquad \tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A\tan B}$

Logarithms

$\log_a x = \frac{\log_b x}{\log_b a}$

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Answer all TEN questions.

Write your answers in the spaces provided.

You must write down all the stages in your working.

1

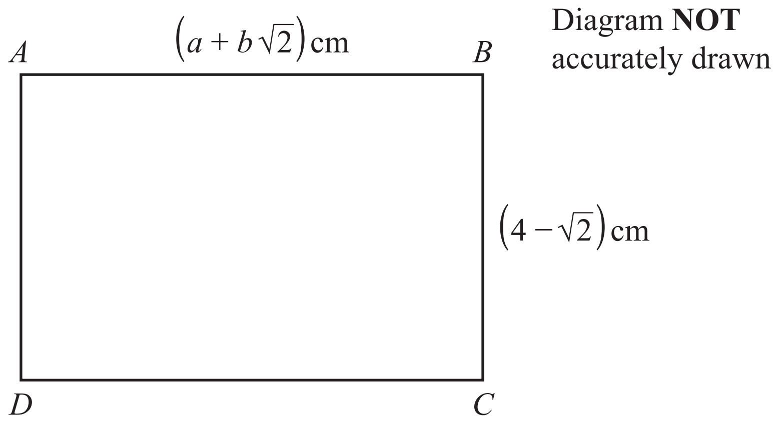

Figure 1

Figure 1 shows the rectangle *ABCD*.

$AD = BC = \left(4 - \sqrt{2}\right)$ cm and $AB = DC = \left(a + b\sqrt{2}\right)$ cm where a and b are integers.

The area of the rectangle is $\left(10 + \sqrt{2}\right)$ cm^2

Find the value of a and the value of b
Show your working clearly.

(4)

(Total for Question 1 is 4 marks)

2 One solution to the following simultaneous equations

$$y = px + 9$$
$$6x^2 - xy = 5$$

is $\left(-\frac{1}{2}, q\right)$, where p is an integer and q is a prime number.

(a) Find the value of p and the value of q

(4)

(b) Hence find the other solution to the equations.

(4)

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Question 2 continued

(Total for Question 2 is 8 marks)

3

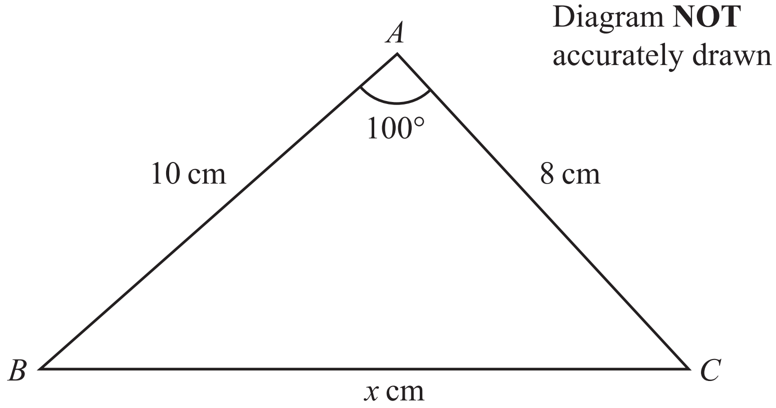

Figure 2

Figure 2 shows triangle ABC where

$$AB = 10 \text{ cm}, \quad AC = 8 \text{ cm}, \quad BC = x \text{ cm} \quad \text{and} \quad \angle BAC = 100^\circ$$

(a) Find, to 3 significant figures, the value of x

(2)

(b) Find, in degrees to one decimal place, the size of

(i) angle ABC (ii) angle ACB

(3)

The bisector of angle ABC meets AC at the point M

(c) Find the area, in cm^2 to 3 significant figures, of triangle BMC.

(4)

Question 3 continued

(Total for Question 3 is 9 marks)

4 (a) Complete the table of values for $y = \frac{x}{2} + 6e^{-2x} + 1$

giving your answers to one decimal place.

x	0	1	1.5	2	3	4	5	6
y	7		2.0			3.0		4.0

(2)

(b) On the grid opposite, draw the graph of $y = \frac{x}{2} + 6e^{-2x} + 1$ for $0 \leqslant x \leqslant 6$

(2)

(c) By drawing a suitable straight line on your graph, obtain estimates, to one decimal place, of the roots of the equation

$$2x + \ln(24 - 5x) = \ln 36$$

(5)

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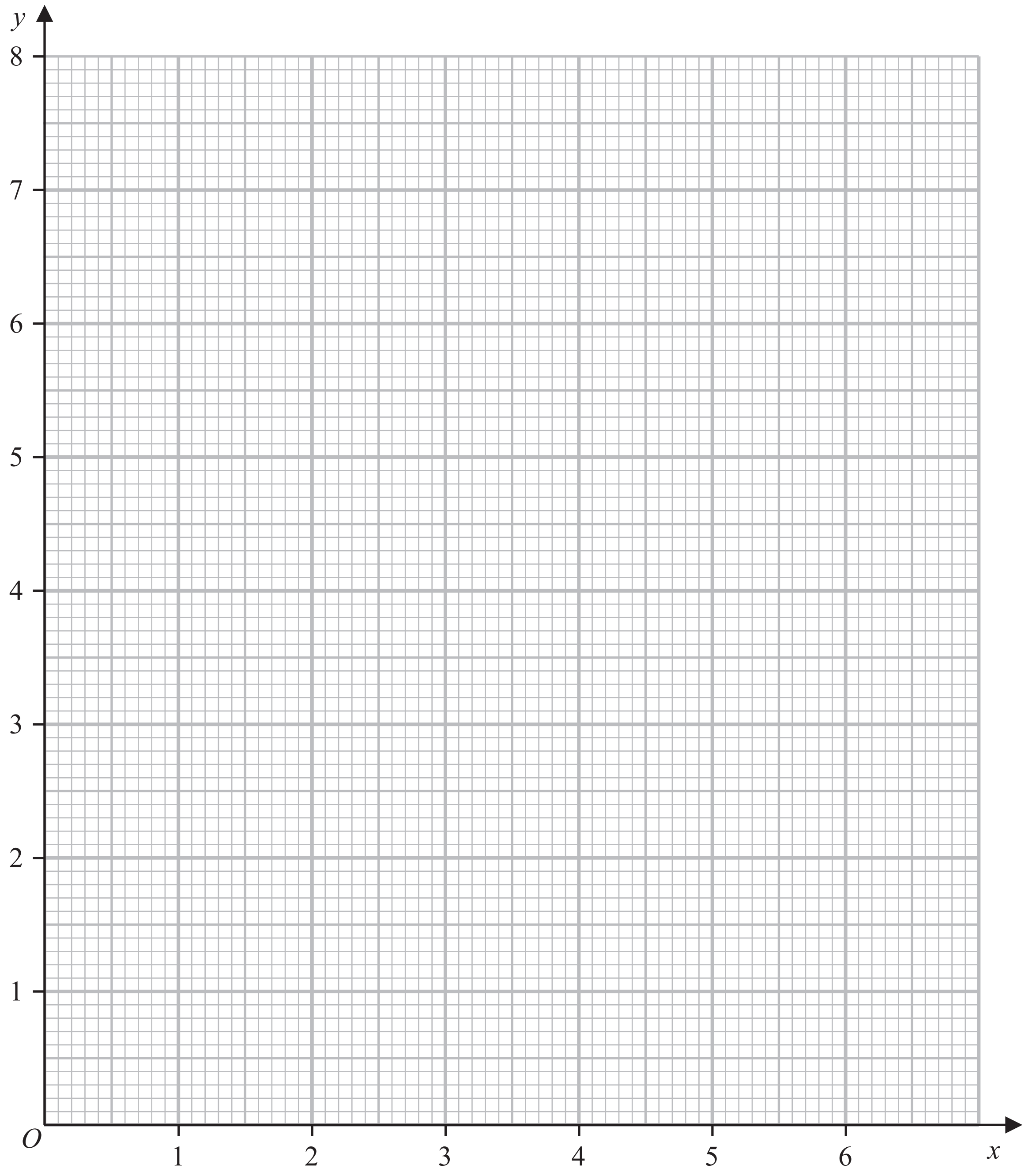
Question 4 continued



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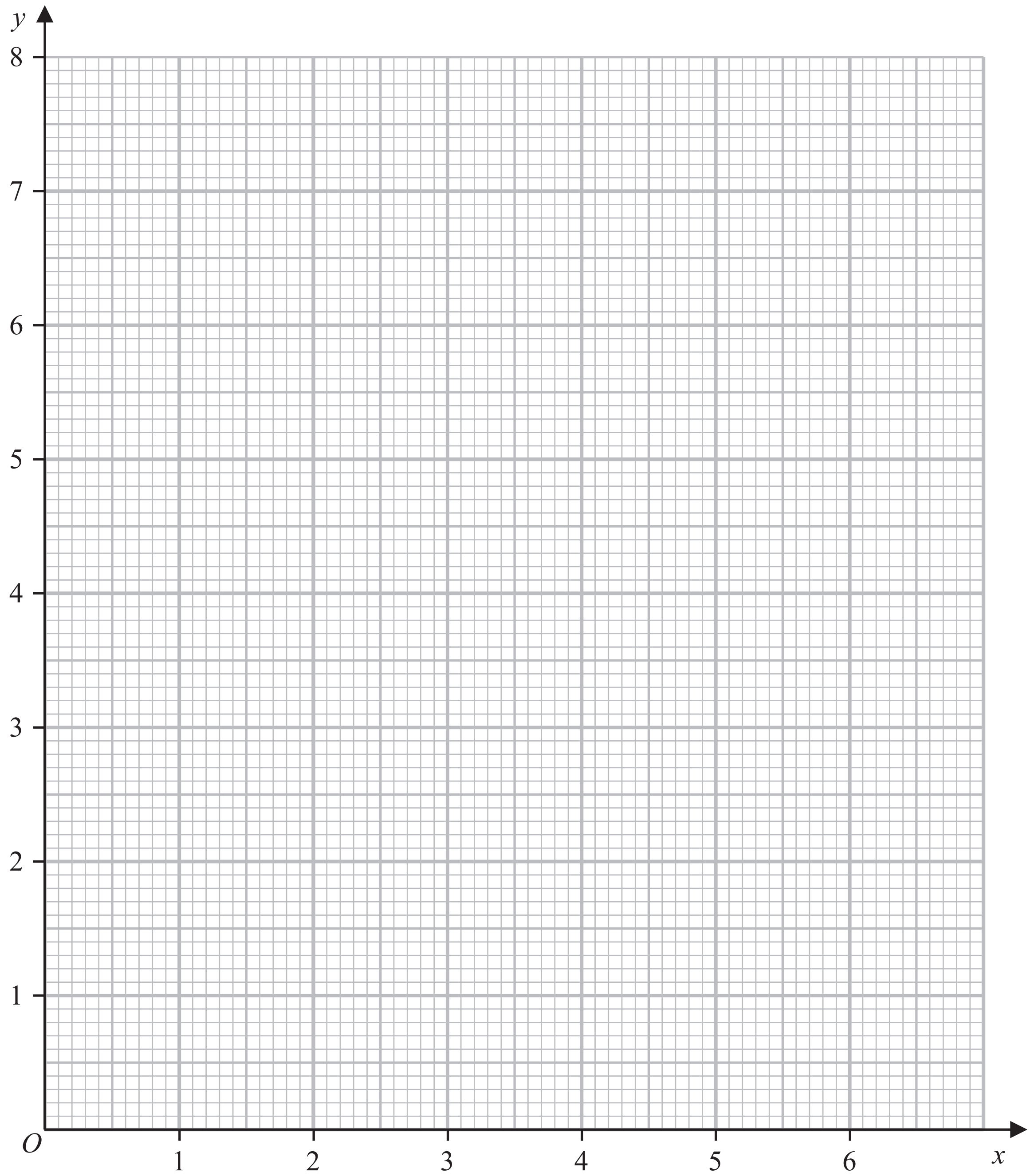
Turn over for a spare grid if you need to redraw your graph.

Question 4 continued

Question 4 continued

Only use this grid if you need to redraw your graph.



(Total for Question 4 is 9 marks)

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Turn over

5 $\mathrm{f}(x) = 2x^3 + ax^2 - 14x + b$ where a and b are constants.

When $\mathrm{f}(x)$ is divided by $(x - 4)$ the remainder is 39

Given that $(x - 1)$ is a factor of $\mathrm{f}(x)$

(a) show that $a = -3$ and find the value of b

(5)

(b) Hence factorise $\mathrm{f}(x)$ completely.

(4)

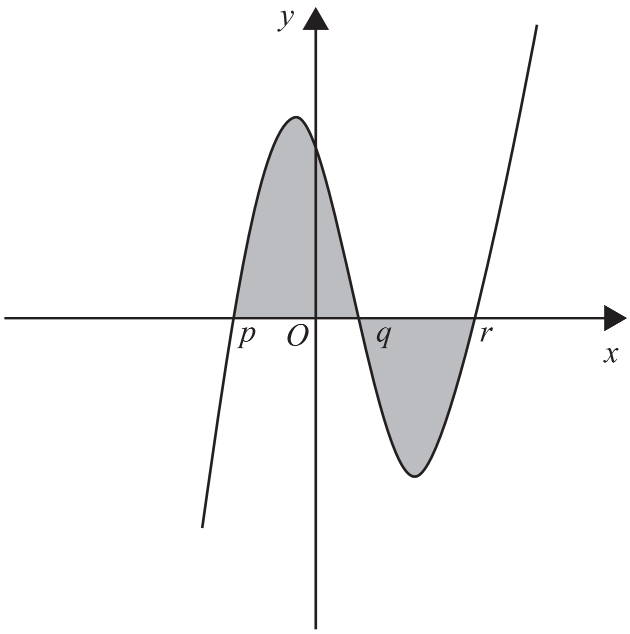

Diagram **NOT** accurately drawn

Figure 3

Figure 3 shows part of the curve C with equation $y = \mathrm{f}(x)$

Given that C crosses the x-axis at the points with coordinates $(p, 0)$, $(q, 0)$ and $(r, 0)$

(c) write down the value of p, the value of q and the value of r

(3)

The region shown shaded in Figure 3 is bounded by the curve and the x-axis.

(d) Use algebraic integration to find the exact area of the shaded region.

(4)

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Question 5 continued

Question 5 continued

Question 5 continued

(Total for Question 5 is 16 marks)

6

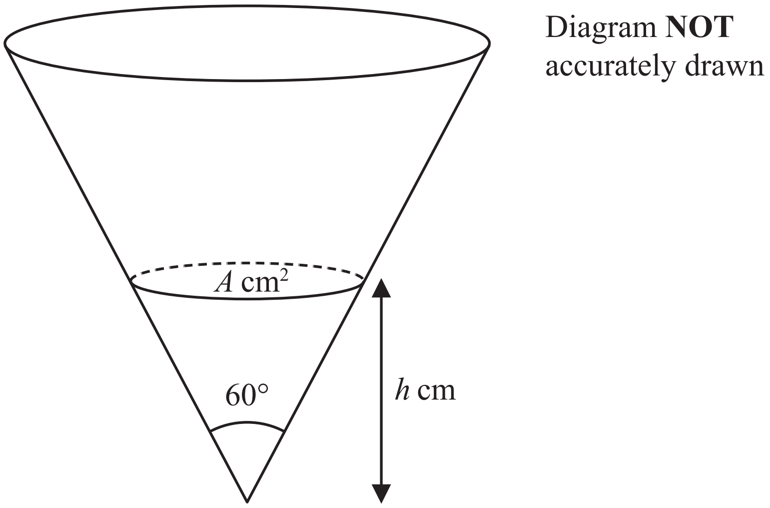

Figure 4

Figure 4 shows a container in the shape of a right circular cone.
The container is fixed with its axis of symmetry vertical.
The vertical angle of the container is 60º as shown in the diagram.

At time t seconds, $t > 0$, the height of oil in the container is h cm and the volume of oil in the container is V cm^3

(a) Show that $V = \frac{1}{9}\pi h^3$

(3)

At time t seconds the surface area of oil in the container is A cm^2, as shown in Figure 4

Oil is dripping out of the bottom of the container at a constant rate of 4 cm^3/s.

(b) Find the exact rate of change, in cm^2/s, of the surface area of oil in the container when $h = 24$

(8)

Question 6 continued

Question 6 continued

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Question 6 continued

(Total for Question 6 is 11 marks)

7 The curve with equation $y = mx^2 + 64\sqrt{x} + 39$ has a stationary point with coordinates $(4, n)$ where m and n are integers.

Using calculus

(a) find the value of m and the value of n

(6)

(b) determine the nature of the stationary point.

(2)

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Question 7 continued

(Total for Question 7 is 8 marks)

8 The nth term of a geometric series G is U_n and the sum of the first n terms of G is S_n

Given that $U_n = \frac{25}{4}\left(\frac{3}{5}\right)^n$

(a) find the exact value of U_5

(1)

(b) Show that $S_n = \sum_{r=1}^{n} \frac{A}{B}\left(\frac{3}{5}\right)^{r-1}$ where A and B are integers to be found.

(3)

The sum to infinity of G is S

(c) Find the least value of n such that $S - S_n < 0.045$

(6)

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Question 8 continued

Question 8 continued

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Question 8 continued

(Total for Question 8 is 10 marks)

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Turn over

9 (a) Expand $(1+2x)^{-\frac{1}{3}}$ in ascending powers of x up to and including the term in x^3 expressing each coefficient as a fraction in its lowest terms.

(3)

(b) Find the range of values of x for which your expansion is valid.

(1)

$$\text{f}(x) = \frac{2 + kx^2}{(1+2x)^{\frac{1}{3}}}$$

(c) Obtain a series expansion of f (x) in ascending powers of x up to and including the term in x^3

Give your coefficients in terms of k where appropriate.

(3)

The coefficient of x^3 in the series expansion of f (x) is $-\frac{8}{3}$

(d) Find the exact value of k

(2)

(e) Hence, using algebraic integration, estimate the value of

$$\int_{0.1}^{0.2} \text{f}(x)\,\text{d}x$$

Give your answer to 4 decimal places.

(5)

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Question 9 continued

Question 9 continued

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Question 9 continued

(Total for Question 9 is 14 marks)

10

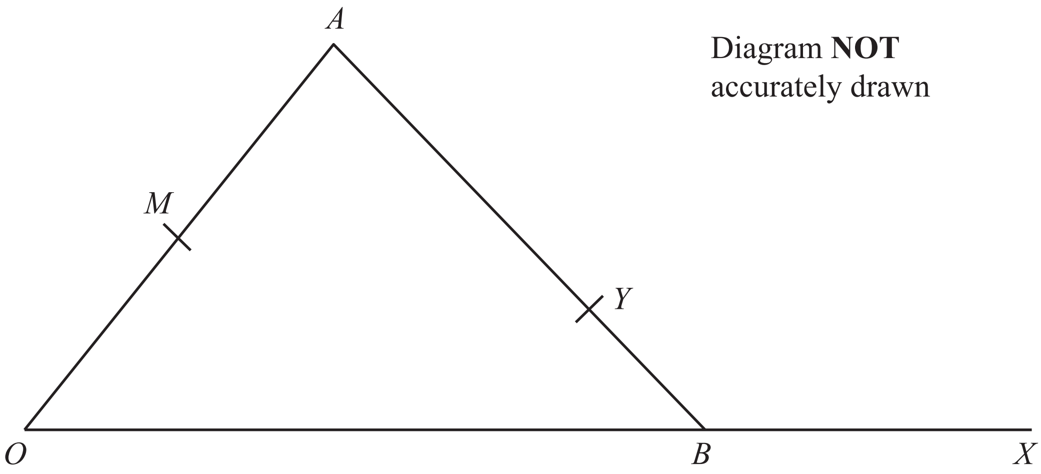

Figure 5

In Figure 5, $\overrightarrow{OA} = 2\mathbf{a}$, $\overrightarrow{OB} = 4\mathbf{b}$ and M is the midpoint of OA.

The point Y lies on AB such that $AY : YB = 3 : 1$

The point X lies on OB produced.

(a) Find as simplified expressions in terms of **a** and **b**

(i) $\overrightarrow{AB}$ (ii) $\overrightarrow{MY}$

(3)

The points M, Y and X are collinear.

(b) Find the ratio $OB : OX$

(5)

(c) Find the ratio of $(\text{Area } \Delta YBX) : (\text{Area } \Delta OAX)$

(3)

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Question 10 continued

Question 10 continued

(Total for Question 10 is 11 marks)

TOTAL FOR PAPER IS 100 MARKS